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On production costs in vertical differentiation models

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Abstract

In this paper, we analyse the effects of the introduction of a unit production cost beside a fixed cost of quality improvement in a duopoly model of vertical product differentiation. Thanks to an original methodology, we show that a low unit cost tends to reduce product differentiation and thus prices, whereas a high unit cost leads to widen product differentiation and to increase prices.

Keywords: fixed cost, variable cost, product quality, vertical differentiation

JEL classification: D21, D43, L13

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1. Introduction

This paper originates from a naïve question: why vertical differentiation models generally assume either a variable cost or a fixed one but, as far as we know, never both? Indeed, models in the line with Mussa and Rosen (1978), Gal-Or (1983), Champsaur and Rochet (1989) and Cremer and Thisse (1994) suppose the development of new quality generates variable costs, whereas those in the line with Shaked and Sutton (1982, 1983) assume that research and development of a new product induces fixed cost. All the same, the improvement of product quality may involve fixed and variable costs. For instance, the development of an automobile model by a car manufacturer or the rise in computer capability, according to the Moore's law, require not only research and development (R&D) fixed costs but also production variable costs.

Obviously, the introduction of a fixed production cost in a model with variable quality cost would not change firms' quality choice. However, would the introduction of a production unit cost in models with fixed quality cost modify firms' quality choice and therefore price choice? Motta (1993) argues that the constant unit production cost can be neglected. In fact, this assumption enables him to strongly simplify the analytical resolution of the game: This is the first answer to our naïve question. But is it really without influences on the firms' choices? Our paper shows that it is not really the case: Thanks to an original analysis methodology, we also achieve to show that the unit production cost plays a specific role in the game by creating an upgrading effect that raises the prices of both products in the same proportion and allows several consumers to substitute the high quality for the low one. As a result, a low unit cost tends to reduce product differentiation and thus prices, whereas a high unit cost leads to widen product differentiation and to increase prices.

2. The model

We consider a differentiated market in which consumers differ in their willingness-to-pay for the best quality of the product. Each consumer buys one unit of the product or none.[1] There are only two identical firms in the industry. Each firm produces one variant of the product and decides on its price.

There is a continuum of consumers whose marginal willingness-to-pay for quality, denoted θ is uniformly distributed over $\left[\underline{\theta},\overline{\theta}\right]$ with a unit density function. When the consumer θ purchases the quality q_i at price p_i, he derives an indirect utility $u(\theta)=\theta q_i - p_i$. Thereby, the consumer $\tilde{\theta}=p_l/q_l$ is indifferent between consuming the low quality product q_l at price p_l or none of the products. The consumer $\hat{\theta}=\left(p_h-p_l\right)/\left(q_h-q_l\right)$ is indifferent between consuming the low quality q_l at price p_l or the high quality q_h at price p_h (with $q_h \geq q_l \geq 0$). As usual with such a model, we assume that the market is not covered ($\underline{\theta}<\tilde{\theta}$), so that the demand for low quality product is $d_l=\hat{\theta}-\tilde{\theta}$ and the demand for high quality product is $d_h=\overline{\theta}-\hat{\theta}$.

[1] Alternatively, one can assume that a consumer derives utility only from the first unit he buys.

In the duopoly, each firm *i* offers one quality q_i and faces a R&D cost $c(q_i)$ that enables the provision of this quality. The quality cost is a standard quadratic function $c(q_i) = \frac{f}{2} q_i^2$ (with $f \geq 0$). Furthermore, firms incur the same unit cost v ($v \geq 0$). Their profits are thus given by:

$$\pi_i = (p_i - v)d_i - c(q_i) \tag{1}$$

The competition between firms takes place in a two-stage game. In the first stage, they decide on the quality q_i to produce. In the second stage, firms choose prices p_i.

3. The game Equilibrium

The game is solved by backward induction in order to provide the subgame perfect equilibrium. In the second stage, firms choose their price taking as fixed qualities q_h and q_l. The maximization of their profits (1) with respect to prices induces the following equilibrium prices:

$$p_h^* = \frac{2\bar{\theta} q_h (q_h - q_l) + 3v q_h}{4q_h - q_l} \qquad p_l^* = \frac{\bar{\theta} q_l (q_h - q_l) + v(2q_h + q_l)}{4q_h - q_l} \tag{2}$$

The corresponding demand functions are:

$$d_h^* = \frac{2\bar{\theta} q_h - v}{(4q_h - q_l)} \qquad d_l^* = \frac{q_h(\bar{\theta} q_l - 2v)}{q_l(4q_h - q_l)} \tag{3}$$

The unit cost *v* tends to increase prices and then to depress demand. The demand for the lowest quality product remains thus positive as long as *v* is sufficiently low in comparison with the willingness-to-pay of the consumer $\bar{\theta}$ for the lowest quality q_l:

$$v \leq \bar{\theta} q_l / 2 \tag{4}$$

In the first stage, firms choose quality specification q_h^* and q_l^* maximizing their profits, according to the following first order conditions:[2]

$$\begin{cases} \dfrac{\partial \pi_h}{\partial q_h} = \dfrac{(2\bar{\theta} q_h - v)\left((4q_h - 7q_l)v + (8q_h^2 - 6q_h q_l + 4q_l^2)\bar{\theta}\right)}{(4q_h - q_l)^3} - fq_h = 0 \\ \dfrac{\partial \pi_l}{\partial q_l} = \dfrac{q_h(\bar{\theta} q_l - 2v)\left((4q_h - 7q_l)\bar{\theta} q_h q_l + (8q_h^2 - 6q_h q_l + 4q_l^2)v\right)}{(4q_h - q_l)^3 q_l^2} - fq_l = 0 \end{cases} \tag{5}$$

In order to simplify these conditions, we operate a first variable substitution by denoting $\lambda \equiv q_h^*/q_l^*$ (with $\lambda \geq 1$). Both conditions (5) induce the following equality:

$$\begin{aligned} &(2\lambda\bar{\theta} q_l^* - v)\left[2(4\lambda^2 - 3\lambda + 2)\bar{\theta} q_l + (4\lambda - 7)v\right] = \\ &\lambda^2(\bar{\theta} q_l^* - 2v)\left[2(4\lambda^2 - 3\lambda + 2)v + (4\lambda - 7)\lambda\bar{\theta} q_l^*\right] \end{aligned} \tag{6}$$

When $v = 0$, it induces, as in Motta (1993), $4\lambda^3 - 23\lambda^2 + 12\lambda - 8 = 0$. The only real solution is $\lambda = 5.2512$. By substituting this value into the first order conditions, we obtain both equilibrium qualities $q_h^* = 0.2533\bar{\theta}^2/f$ and $q_l^* = 0.0482\bar{\theta}^2/f$.[3]

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[2] The second order conditions are studied in appendix A1.

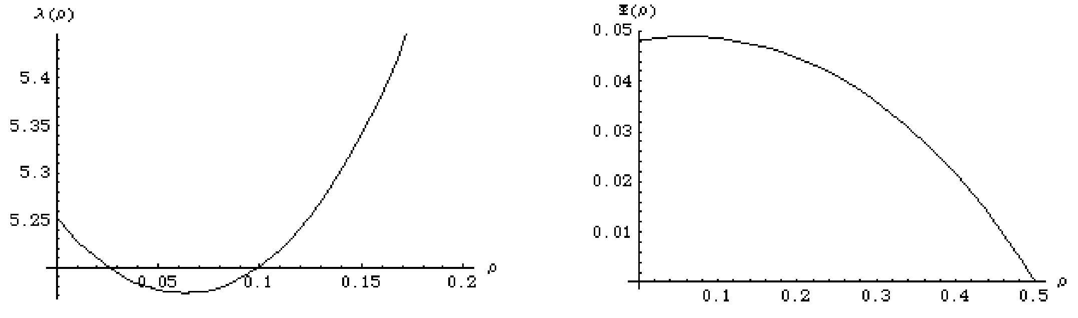
[3] Motta (1993) proves that these qualities are indeed Nash equilibrium.

When $v > 0$, we cannot analytically express the equilibrium qualities. We can also turn to numerical simulations (see section 4) or carry out an *ex post* analysis of the equilibrium. To this aim, we operate a second variable substitution by expressing the unit cost *v* in term of a percentage ρ of the *ex post* maximal willingness-to-pay for the equilibrium low quality $\bar{\theta} q_l^*$ ($\rho \equiv v / \bar{\theta} q_l^*$, with $\rho \in \left[0, \frac{1}{2}\right]$). Since ρ is introduced into the analysis *after* the implicit determination of q_h^* and q_l^* in (5), it corresponds to a *variable substitution* and not to an endogenisation of *v*. Thanks to this variable substitution, the following proposition holds.

Proposition 1. *When the unit cost v is expressed in term of a percentage ρ of the ex post maximal willingness-to-pay the low quality* $\bar{\theta} q_l^*$ *and* $\rho \in \left[0, \frac{1}{2}\right]$, *the differentiation parameter is the only one real root* $\lambda(\rho)$ *of the polynomial function defined by*

$$P(\lambda;\rho) = 4\left(1 - 4\rho^2\right)\lambda^4 + \left(12\rho^2 + 8\rho - 23\right)\lambda^3 + 4(1+\rho)(3-2\rho)\lambda^2 + 4\left(\rho^2 + 2\rho - 2\right)\lambda + 4\rho - 7\rho^2 \tag{7}$$

Graph 1. Optimality conditions of the quality game



Graph 1a. ***λ(ρ)*** **Graph 1b.** ***Φ(ρ)***

For each value of $\rho, P(\lambda;\rho)$ has one real root $\lambda(\rho)$ greater than unity, which decreases with ρ and reaches its minimum for $\lambda(0.07) = 5.1741$, before increasing and then exceeding its initial value for ρ =0.13 (cf. graph 1a).[4] For each value of ρ, we thus compute the only associated value $\lambda(\rho)$ and then substitute $\lambda(\rho) q_l^*$ for q_h^* and $\rho \bar{\theta} q_l^*$ for v in the second optimality condition (5). We therefore deduce the proposition 2 below.

Proposition 2. *When the unit cost v is expressed in term of a percentage ρ of the ex post maximal willingness-to-pay the low quality* $\bar{\theta} q_l^*$ *and* $\rho \in \left[0, \frac{1}{2}\right]$, *the equilibrium low quality* q_l^* *can be expressed as:*

$$q_l^* = \Phi(\rho).\bar{\theta}^2 / f \tag{8}$$

$$\textit{with } \Phi(\rho) = \frac{(1-2\rho)\lambda(\rho)\Big(\big(4.\lambda(\rho) - 7\big)\lambda(\rho) + 2\big(4\lambda(\rho)^2 - 3\lambda(\rho) + 1\big)\rho\Big)}{\big(4.\lambda(\rho) - 1\big)^3} \tag{9}$$

and the equilibrium high quality is defined by $q_h^* = \lambda(\rho) q_l^*$.

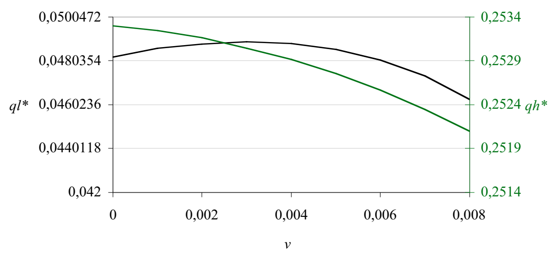
[4] Calculations and simulations were made with the software Mathematica. We cannot express straightforwardly the form of this root.

The function $\Phi(\rho)$ slightly increases up to $\Phi(0.06) = 0.049$ before decreasing to zero for $\rho = 0.5$ (cf. graph 1b). When $\rho = 0$, the game solution corresponds to the Motta's quality equilibrium without unit cost. The game equilibrium formulation set out in the proposition 2 allows us to sharpen the effects of key parameters of the game on firms' choices, particularly the production unit cost.

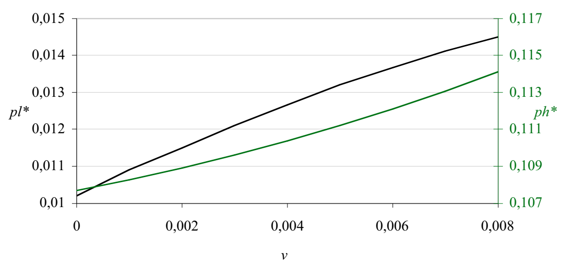
4. The effects of variable cost introduction

In order to analyse the effects of the existence of a production unit cost, we examine the game equilibrium for different values of v and draw a parallel between these simulations and the analytical expression of the equilibrium (see appendix A2 for a simulation exemple). We restrict here the study to the duopoly case.

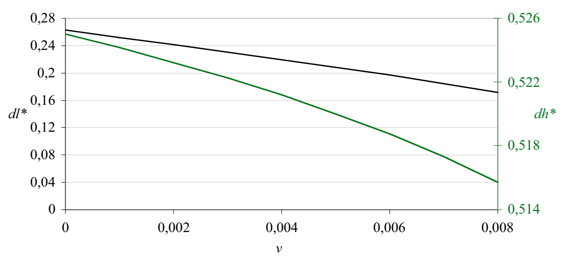
Graphs 2. Effects of a rise in unit cost (for $\overline{\theta}^2 = 1$ and $f = 1$)



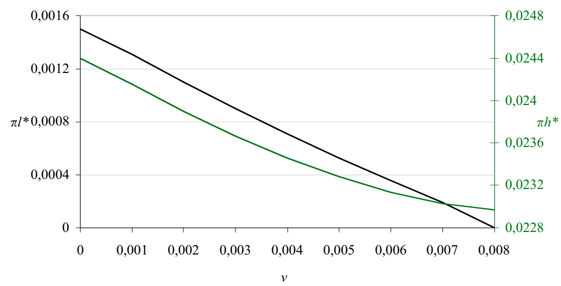
Graph 2a. Qualities



Graph 2b. Prices



Graph 2c. Demands



Graph 2d. Profits

According to the graph 2a, a unit cost lower than 0.003 tends to improve the low quality until $q_l^* = 0.049$ whereas a higher unit cost reduces it. That is in the line with the shape of $\Phi(\rho)$ which reaches its maximum for $\rho = 0.06$, such as $\Phi(0.06) = 0.049$. Furthermore, the unit cost tends to downgrade the high quality. This effect arises from its contradictory impact on q_l^* and $\lambda(\rho)$. The latter decreases for $\rho \leq 0.07$, for which q_l^* grows (with $\Phi(0.07) \approx \Phi(0.06)$), and increases when q_l^* falls. Simulations show that the decreasing effect outweighs the increasing one.

Moreover, the unit cost weighs on prices, through its direct effect on production cost and its indirect impact on product differentiation, beyond the threshold 0.07 (graph 2b). A higher unit cost also leads to a loss in demand addressed to each firm (graph 2c).

Noticeably, profit of both firms decreases with the unit cost, in such a way as a high unit cost implies a high quality monopoly (graph2d).

The effects of the production unit cost may also be resume in the proposition below.

Proposition 3. *A low unit cost induces an improvement of the low quality (if $\rho \leq 0.06$), a degradation of the high quality, a fall in product differentiation (if $\rho \leq 0.07$) and an increase in prices. A high unit cost induces a degradation in the low quality (if $\rho \geq 0.06$) and in the high quality, an increase in product differentiation (if $\rho \geq 0.07$) and an increase in price. Beyond the unit cost threshold $\bar{v} = 0.18\,\bar{\theta} q_l^*$ ($\rho \geq 0.18$), the low quality product firm is ousted.*

Furthermore, equations (8) and (9) enable us to give some analysis elements for other key parameters of the firms' choice. In order to carry out this study, we present below an illustration of the game equilibrium with a unit cost $v = 0.1\bar{\theta} q_l^*$. Following the equation (8), the differentiation parameter is here $\lambda(0.1) = 5.2015$ and the equilibrium is characterized by:

$$\begin{aligned} q_h^* &= 0.2528\,\bar{\theta}^2/f & q_l^* &= 0.0486\,\bar{\theta}^2/f \\ p_h^* &= (0.0038 + 0.1072\bar{\theta})\bar{\theta}^2/f & p_l^* &= (0.0028 + 0.0103\bar{\theta})\bar{\theta}^2/f \\ d_h^* &= 0.5252\bar{\theta} - 0.0050 & d_l^* &= 0.2626\bar{\theta} - 0.0525 \end{aligned} \tag{10}$$

According to (10), the *fixed cost parameter f* tends to decrease product differentiation ($q_h^* - q_l^* = 0.207\,\bar{\theta}^2/f$) and prices, through deterioration of the quality of both products. Demand for each variant is independent of *f* which doesn't affect the quality-adjusted prices p_i^*/q_i^*. Obviously, the more the maximal marginal willingness-to-pay $\bar{\theta}$ is high, the more qualities, prices and profits are great.

5. Conclusion

The introduction of a positive unit production cost in a vertical differentiation model with fixed cost for quality improvement is not neutral. Thanks to an original analysis method of the game equilibrium, we highlight that a low unit cost tends to reduce product differentiation and thus prices, whereas a high unit cost leads to raise product differentiation and prices. Furthermore, a high unit cost may lead to a high quality monopoly.

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Appendix 1. Second order conditions of the quality subgame

The first equation of conditions (5) is decreasing with q_h:

$$\frac{\partial^2 \pi_h}{\partial q_h^2} = -\frac{8\left(\bar{\theta} q_l - 2v\right)}{\left(4q_h - q_l\right)^2}\left[v\left(2q_h - 5q_l\right) + \bar{\theta} q_l\left(5q_h + q_l\right)\right] - f < 0$$

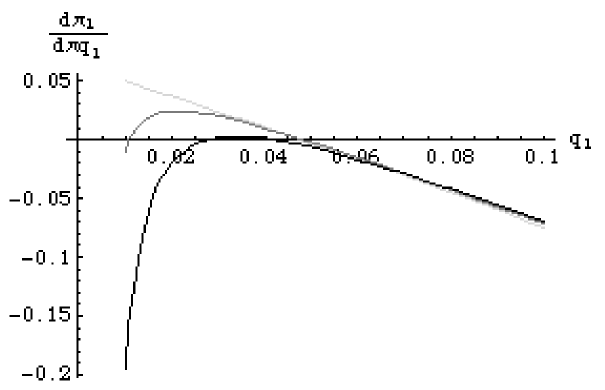
because the first term is positive and the term in brackets is negative, and $\left.\frac{\partial \pi_h}{\partial q_h}\right|_{q_h=0} = \frac{v\left(4\bar{\theta} q_l - 7v\right)}{q_l^2} > 0$ when the condition (4) is met.

The second equation of (5) is increasing for low values of q_l and decreasing for larger values of q_l. The second derivative of firm l's profit is:

$$\frac{\partial^2 \pi_l}{\partial q_l^2} = \frac{2q_h\left[4\left(16q_h^3 - 16q_h^2 q_l + 6q_h q_l^2 - 3q_l^3\right)v^2 - \left[\bar{\theta} q_h\left(8q_h + 7q_l\right) - 4v\left(5q_h + q_l\right)\right]\bar{\theta} q_l^3\right]}{\left(4q_h - q_l\right)^2 q_l^3} - f$$

A solution exists if there is $x > 0$ such as $\partial^2 \pi_l(x)/\partial q_l^2 = 0$ and $\partial \pi_l(x)/\partial q_l > 0$. Some numerical simulations show that a stable equilibrium exists (graph. A1) if the unit cost is lower than a threshold $\hat{v}$, which is all the more high as $\bar{\theta}$ is high (for $\bar{\theta} = 1$ and f=1, $\hat{v}$=0.0108).

Graph A1. Simulation of $\partial \pi_l / \partial q_l$ for $q_h = 0.25$*



* *For ρ=0 (clear grey curve), 0.005 (dark grey curve) and 0.01 (black curve)*

A2. Simulations results for $\overline{\theta} = 1$ and $f = 1$.

Tab. A1 Direct simulations of first order conditions

v	0	0.002	0.004	0.006	0.008
q_l^*	0.0482	0.0488	0.0488	0.0481	0.0463
q_h^*	0.2533	0.2532	0.2529	0.2525	0.2521
p_l^*	0.0102	0.0115	0.0126	0.0137	0.0145
p_h^*	0.1077	0.1089	0.1104	0.1121	0.1141
d_l	0.2625	0.2412	0.2196	0.1970	0.1714
d_h	0.5250	0.5232	0.5223	0.5188	0.5157
π_l	0.0015	0.0011	0.0009	0.0003	0
π_h	0.0244	0.0239	0.0235	0.0231	0.0230

Tab. A2 Simulations of first order conditions with $v \equiv \rho\overline{\theta}q_l^*$

ρ	0	0.05	0.1	0.15	0.18
$\Phi(\rho)$	0.0482	0.0489	0.0486	0.0472	0.0459
$\lambda(\rho)$	5.2512	5.1765	5.2015	5.3397	5.4893
q_l^*	0.0482	0.0489	0.0486	0.0472	0.0459
q_h^*	0.2533	0.2531	0.2528	0.2523	0.2520
v	0	0.0024	0.0048	0.0071	0.0082